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Executive Summary

The Diversification Return has long been supposed to represent the incremental return associated with portfolios that are regularly rebalanced compared to those that are not. In this paper, we test the hypothesis that it can be correctly associated with a “rebalancing premium” for various factor portfolios. We then go on to determine how much of the excess return of these factor portfolios may be attributed to the diversification return, and therefore whether or not, a traditional factor-based explanation of their performance is more appropriate.

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1. Introduction

The notion of “diversification return” has been widely discussed by academics and practitioners over the years [1, 2, 3, 4, 5, 6]. The consensus is that it represents the incremental return associated with portfolios that are regularly rebalanced, compared to those that are not, and consequently, it is often referred to as the “rebalancing premium.”

The idea first surfaced with Fernholz and Shay [1], who used the methods of Stochastic Portfolio Theory to derive an expression for the log-return of a rebalanced portfolio, in terms of the weighted average of individual stock log-returns and an “excess growth rate.” Later, Booth and Fama [2] provided an independent derivation of this relationship and furthermore coined the term “diversification return” for the contribution they attributed to the benefits of portfolio diversification. Both sets of researchers express the diversification return as half the difference between the weighted average stock volatility of the portfolio’s constituents and the portfolio’s volatility. Hence its contribution is positive and can be readily interpreted as arising from the risk reduction benefit of holding a portfolio rather than individual stocks.

The derived relationship is elegant, but not universal, as it requires that a portfolio is rebalanced back to a constant (unchanging) set of portfolio weights. This is quite a restrictive condition as, in most cases, portfolio weights will vary from rebalance to rebalance (for example, to counter decaying factor exposures, or to reduce portfolio risk under changing market conditions). This, however, has not prevented subsequent attempts to widen the scope of the relationship and to draw conclusions about how rebalancing and diversification relates to the return of more general portfolios [5]. Indeed, the concept has even been applied to several smart beta portfolios, leading to the conclusion that the diversification return is the sole source of their excess return [3, 4].

To evaluate the true nature of the diversification return and its role in the performance of factor portfolios, we will construct portfolios that strictly adhere to the conditions under which the diversification return has previously been specified. That is, we will construct portfolios that are regularly rebalanced back to the same set of weights and then examine whether their performance properties are in line with expectations. It is instructive to compare these rebalanced portfolios to the equivalent non-rebalanced portfolios; this allows us to tease out the effect of rebalancing and address the question whether rebalancing is always advantageous. In order to fully realize this comparison, we extend the definition of diversification return to one that is applicable to the non-rebalanced portfolios. Furthermore, we will examine how significant a contribution of the diversification return is to our portfolios’ performance, and compare it to other more traditional sources of excess return.

The structure of this note is as follows. In Section 2, we present a definition of the “diversification return” that may be applied to both rebalanced and non-rebalanced portfolios. Using this as a starting point, we also derive an approximate result, which we believe is the basis for the widespread belief that “rebalancing is best.” In Section 3, we construct several simple factor portfolios that rebalance back to fixed weights, and compare their performance to equivalent non-rebalanced portfolios. In Section 4, we decompose the geometric return of our rebalanced portfolios into the diversification return and the weighted average stock geometric return (or “strategic return”). We apply this decomposition to determine the dominant contributor to each portfolio’s absolute and relative returns and specify how this relates back to the factor exposure of our portfolios. In Section 5, we draw our conclusions.

2. A general definition of diversification return

Fernholz and Shay's formula for the diversification return is derived for a portfolio that is regularly rebalanced back to a fixed set of weights. In this section, we will propose an alternative definition that may be applied to more general portfolios, irrespective of whether they are rebalanced or not. The advantage of employing this broader definition is that it will allow us to compare the diversification return of rebalanced with non-rebalanced portfolios in section 3. Importantly, we show that for portfolios that are rebalanced back to a fixed set of weights, this more general expression reduces to Fernholz and Shay's formula. To arrive at this general definition of diversification return, we briefly review the definitions of geometric mean return, arithmetic mean return and the relationship between them.

Consider a time series of returns to a stock or portfolio. Given the set of time periods $t = 0, 1, \dots, T$, we can define the geometric mean return of a portfolio (stock) by:

$$g = \sqrt[T]{(1 + r_1) \times (1 + r_2) \times \dots \times (1 + r_T)} - 1 \quad (1)$$

where r_t is the return of the portfolio (stock) over the t^{th} time period. The arithmetic mean return is:

$$\bar{r} = \frac{1}{T}(r_1 + r_2 + \dots + r_T) \quad (2)$$

A useful approximate relationship [2, 7] exists between these two quantities:

$$g \approx \bar{r} - \sigma^2/2 \quad (3)$$

where σ is the volatility of the returns. In other words, the geometric mean return is expected to be smaller than the arithmetic mean return by half the value of the volatility of the returns.

In line with several authors [5, 7], we define the diversification return (DR) of *any* portfolio, irrespective of whether it is rebalanced back to its original weights or not, as the difference between the geometric mean return of the portfolio G_p and a weighted average of the geometric mean returns of individual stocks g_i . That is:

$$DR = G_p - \sum_{i=1}^N W_i * g_i \quad (4)$$

where W_i are a suitably defined set of individual stock weights. The subtracted term is often referred to as the "Strategic Return".

This definition of diversification return is clearly at odds with the idea that rebalancing is a necessary condition for the diversification return to exist. Indeed, some authors have argued that “true” diversification return results from the selling of stocks that have gained in weight and buying those that have reduced in weight in a way that is natural in the rebalancing process [6]. We take a practical view that since it can be defined and measured for both rebalanced and non-rebalancing portfolios, then a comparison can be legitimately made, regardless of any difference in the “return mechanics,” explaining each type of portfolio’s performance.

However, an ambiguity remains in this definition for portfolios, whose weights are not held constant at the beginning of each time period; whether this is achieved by actively rebalancing back to the set of weights defined at $t = 0$, or simply through fortunate price movements. Such a portfolio’s weights will, in general, differ from the initial set of weights, because of price drift, portfolio additions and deletions or simply through rebalancing to some other distinct set of weights. What set of weights should be used in this case?

Some authors [7] have addressed this problem by proposing that an average of changing, or drifting weights, should be used in (4). For our purposes however, we will restrict our attention to portfolios rebalanced to fixed weights or else portfolios that are not rebalanced and lack stock additions or deletions. In either case, the initial set of weights at $t = 0$ can be used for the weights W_i in equation (4).

As a mere definition, equation (4) is not very enlightening as a characterization of portfolio performance. However, it is possible to show that this expression reduces to the formulae of Fernholz and Shay [1] and Booth and Fama [2] in the case of a portfolio that is rebalanced to a fixed set of weights. Instead of reproducing their analysis, we will follow Willenbrock [6], whose analysis leads directly to an estimate for equation (4) in terms stock and portfolio level volatility.

For portfolios rebalanced to a fixed set of weights, Willenbrock notes that their arithmetic mean return is exactly equal to a weighted average of the arithmetic mean returns of the individual stocks (see Appendix), that is:

$$\bar{R}_p = \sum_{i=1}^N W_i * \bar{r}_i \quad (5)$$

Through equation (3), both the portfolio and stock arithmetic mean returns in equation (5) can be expressed the sum of the geometric mean return plus the half the volatility, so that:

$$G_p + \frac{1}{2} \sigma_p^2 \approx \sum_{i=1}^N W_i * (g_i + \frac{1}{2} \sigma_i^2) \quad (6)$$

where σ_i are individual stock volatilities and σ_p is the portfolio volatility. Rearranging equation (6) leads directly the following approximate expression for the diversification return:

$$DR \approx \frac{1}{2} \left[\sum_{i=1}^N W_i * \sigma_i^2 - \sigma_p^2 \right] \quad (7)$$

This is precisely Fernholz and Shay’s expression. It has been interpreted by many authors as expressing the benefits of diversification, since it represents the difference between the weighted average of individual stock volatilities and the portfolio volatility. This is also expected to be positive, since a portfolio’s volatility will be on average lower than that of its constituent stocks [7].

In the case of a non-rebalancing portfolio, we can show that the general definition for the diversification return given by equation (4) is also positive using Jensen’s inequality [11]. Hence, rebalancing is not necessary to earn a positive

diversification return, at least as it has been defined here. Notwithstanding this, one may still ask how does it compare in magnitude to that of the rebalanced portfolio estimate given by equation (7)?

By way of an answer, we note that, although equation (5) is not strictly satisfied by the non-rebalanced portfolio, it may still approximately hold true. This would clearly be the case whenever the portfolio weights do not drift too far from their initial values. Hence, for suitably diverse portfolios, and during “normal” market conditions, equation (7) should also provide a reasonable estimate of the diversification return of both a rebalanced and non-rebalanced portfolio.

This reasoning would seem to undermine the argument deployed by some authors, that rebalancing tends to improve long-term returns [8]. However, if we could prove that equation (7) represents an upper bound to the diversification return of a non-rebalanced portfolio, then we could conclude that rebalancing is always advantageous. We are not aware of such a theoretical proof, so instead we will test this empirically in Section 3.

3. Rebalanced versus non-rebalanced factor portfolios

In this section, we construct several portfolios based on the FTSE USA Index of stocks. We begin by defining a set of portfolio weights for each September between 2000 and 2020, with the condition that the portfolio only contains stocks which *survive* the whole year to the following September. Our “non-rebalance” portfolio is then the set of weights that drift freely with price movements from September to September. For our “rebalance portfolio”, we use the same set of September weights, but also define the weights (on the third Friday) of every other month, which are the same as the preceding September weights. Hence, during the September to August period of each year, we define a portfolio whose weights are set back to a constant set of weights on a monthly basis. This “rebalance portfolio” therefore satisfies sufficient conditions that Fernholz and Shay’s expression (7) for diversification return is valid.

Note that our “non-rebalance” portfolio is rebalanced on an annual basis. This is necessary, as if we were to impose the stock survivorship requirement over the entire period from 2000 to 2020, the resulting portfolio would consist of relatively few stocks and less diversified outcomes. Additionally, the factor exposures of such portfolios would decay so that, for example, a portfolio with value exposure in September 2000 would not necessarily exhibit value exposure in September 2020. Our aim is to compare results for diversified portfolios with fixed factor characteristics on a year-by-year basis.

We will construct five sets of rebalance and non-rebalance portfolios; three factor tilt portfolios representing Quality, Value and Low Volatility, one equal weight portfolio and a market-capitalization weighted portfolio. In each case, the eligible universe of stocks constitutes the September to September surviving stocks as outlined above.

The factor tilt portfolios are constructed using the tilt equation:

$$W = M * S \tag{8}$$

where M is a vector of market capitalization weights, S is a positive score that varies monotonically with factor score and the factor weights W are normalized to sum to one. The tilt is applied to the entire universe, and then the resulting portfolio restricted to surviving stocks. These portfolios are relatively pure from the factor perspective, with strong targeted exposure and relatively weak off-target exposures. For more details about this construction and factor definitions see [9].

Note that we have not included tilt portfolios representing Momentum or Size in our analysis. In the case of Size, this is simply because the equal weight portfolio acts as a good proxy for the Size factor. In the case of Momentum, unlike other factors, frequent rebalancing is necessary to overcome the rapid decay in factor exposure. Typically, we find that a non-rebalanced portfolio with positive momentum exposure at the beginning of a twelve-month period will have a negative exposure by the end of that period. In other words, a non-rebalanced Momentum portfolio is somewhat of an oxymoron, so we exclude it from this analysis.

The basis point difference in monthly geometric mean return between the rebalanced and non-rebalanced portfolios is displayed in Table 1 for each portfolio, and for each annual period ending in September of the given year. There are periods where the rebalanced portfolios perform better than the non-rebalanced portfolios, while for other time periods the reverse is true. Importantly, neither approach dominates over the entire period for any of our portfolios. Moreover, the differences are only a few basis points for most years, apart from 2001, where the non-rebalance portfolios perform significantly better, and in 2009, where the rebalance portfolios exhibit a significant advantage.

Table 1: Difference in mean monthly geometric return between Rebalanced and Non-Rebalanced portfolios (bps)

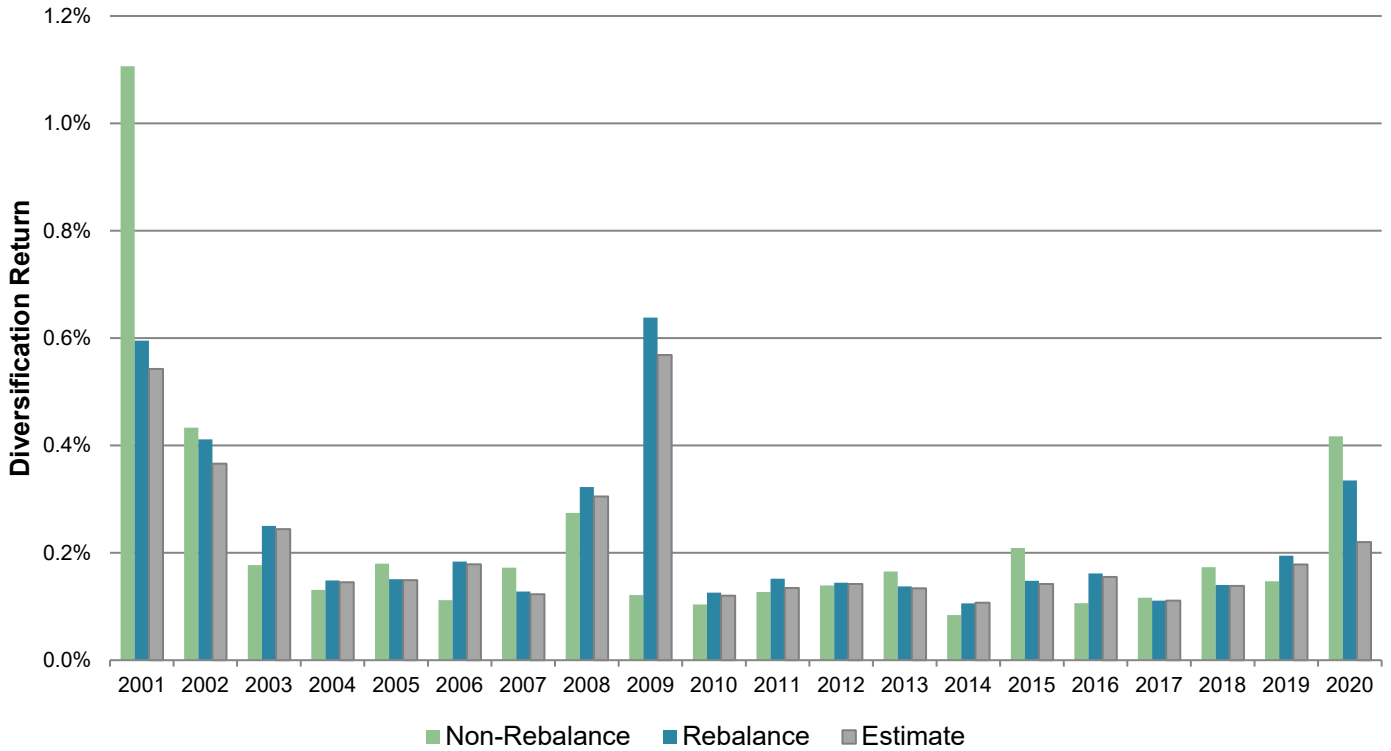
	Market Cap	Equal Weight	Quality	Low Volatility	Value
2001	-74	-30	-51	-36	-41
2002	-4	-5	-2	-3	-5
2003	7	4	7	9	11
2004	2	1	2	2	2
2005	-3	-5	-3	-4	-7
2006	6	5	7	6	6
2007	-4	-7	-4	-2	-5
2008	-3	5	5	-6	-6
2009	78	104	52	63	90
2010	2	4	2	3	4
2011	0	1	2	2	-2
2012	2	1	1	2	3
2013	-3	-1	-3	-2	-2
2014	3	2	2	2	1
2015	-5	-10	-6	-3	-6
2016	6	9	6	5	7
2017	0	-1	-1	0	-1
2018	-3	1	-3	-3	-1
2019	5	3	5	5	5
2020	-7	4	-8	-1	-2

Source: FTSE Russell. Data based on the FTSE USA Index between September 2000 and September 2020. Please see the end for important legal disclosures.

As explained in section 2, we have defined the strategic returns of this set of rebalanced and non-rebalanced portfolios to be identical. Therefore, we are led to the conclusion that the differences in geometric return are caused by differences in diversification return.

Figure 1 shows the diversification return of our rebalanced and non-rebalanced quality portfolios for each year ending in September, from 2001 to 2020. We have also included a single bar to represent the estimated diversification return given by equation (7), which we have calculated using the volatility of the rebalanced portfolio. This single bar is appropriate since it is easy to verify that the volatility of the rebalanced and non-rebalanced portfolios are similar during each annual period.

Figure 1: Diversification Return of Rebalanced and Non-Rebalanced Quality Portfolios

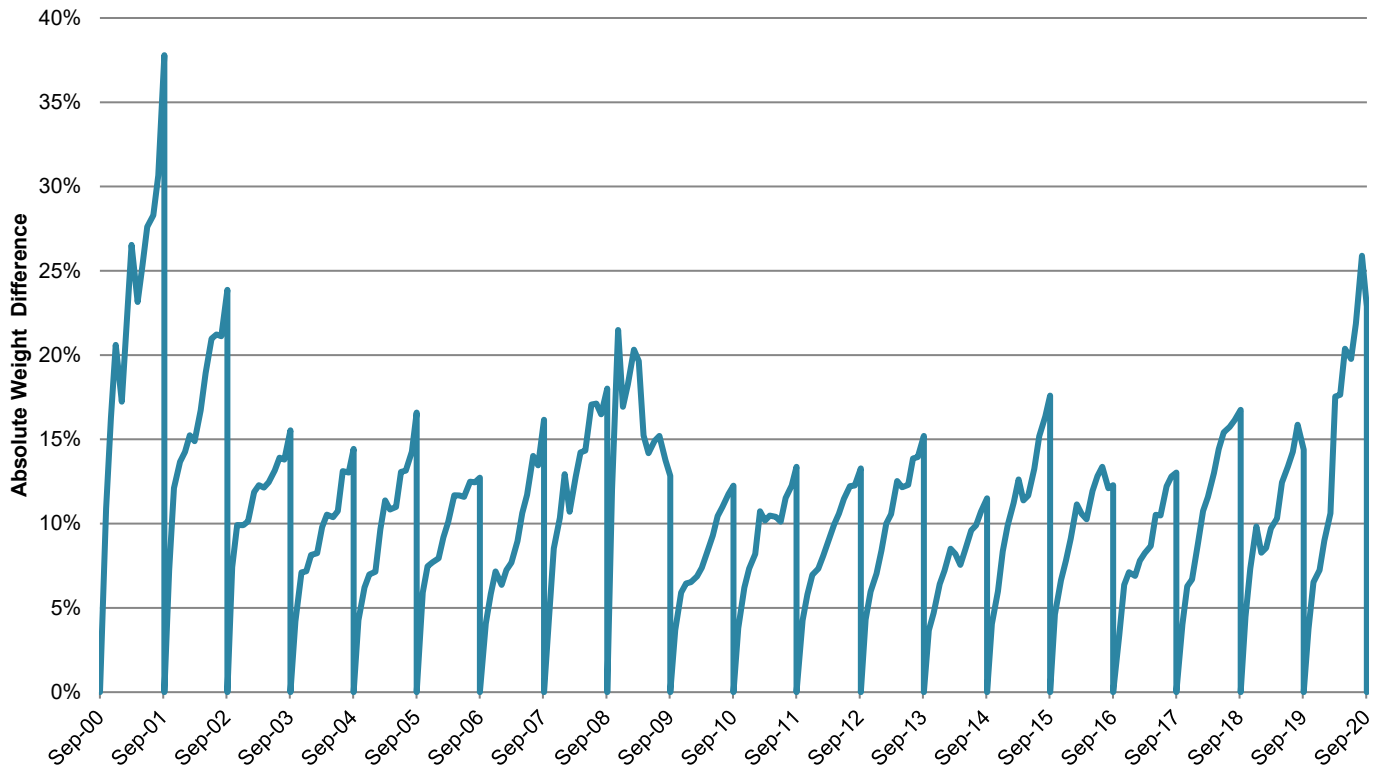


Source: FTSE Russell. Data based on the FTSE USA Index between September 2000 and September 2020. Please see the end for important legal disclosures.

The first thing to note is that the diversification return of the rebalanced and the non-rebalanced portfolios are remarkably similar to one another for all years, with the exception of 2001 and 2009. In the aftermath of the technology bubble of 2000-2001, the non-rebalance portfolio shows a greater diversification return. Conversely, in 2009, the financial crisis favored the diversification return of the rebalance portfolio. The common feature is that they represented periods of extreme market dislocation, where the non-rebalanced portfolio could move significantly away from its initial set of portfolio weights.

We have already argued that under normal circumstances we would expect equation (7) to be a good proximation of both the rebalance and non-rebalance diversification returns. It is apparent that the rebalance portfolio's diversification return is well fitted by equation (7) throughout the study period (apart from the high volatility period of 2020). However, for the non-rebalance portfolio in the years 2001, 2009 and 2020, equation (7) is not a particularly good estimate. We expected that the estimate would be reasonable for the non-rebalanced portfolios during periods when the drifting of weights is not too extreme. This is confirmed by figure 2, where we have plotted the absolute weight difference between the initial portfolio and the drifted weights of the non-rebalance portfolio through time.

Figure 2: Absolute Weight Difference of Drifted Weights for Non-Rebalanced Quality Portfolio



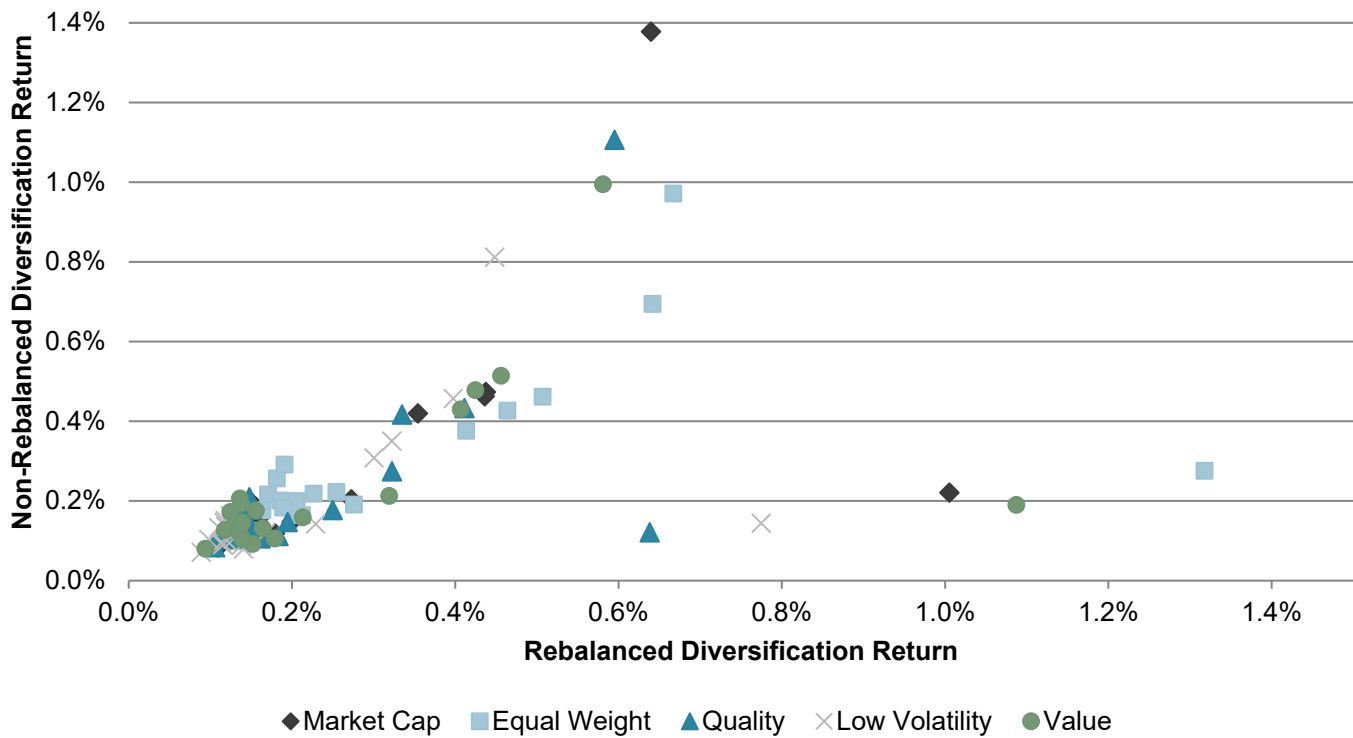
Source: FTSE Russell. Data based on the FTSE USA Index between September 2000 and September 2020. Please see the end for important legal disclosures.

Clearly in most years, the weights do not drift more than by about 15% from their initial values, but in the years 2001, 2009 and 2020, they are above 20%. These are the years where the estimate of the diversification return from equation (7) is poor.

Therefore in “normal years,” equation (7) gives a good estimate for both the rebalanced portfolio *and* the non-rebalanced portfolio diversification returns. However, this estimate is poor for the non-rebalanced portfolio, when there is a lack of mean-reversion in the returns of individual stocks, which inevitably leads to weight concentration (dilution) in the better (worse) performing stocks.

Figure 3 is a scatter plot of the diversification return of the rebalanced portfolios versus the non-rebalanced portfolios.

Figure 3: Scatter plot of diversification return of Rebalanced versus Non-Rebalanced portfolios



Source: FTSE Russell. Data based on the FTSE USA Index between September 2000 and September 2020. Please see the end for important legal disclosures.

With the exception of a few outliers representing the volatile periods, the points cluster around the 45-degree line which implies that, under normal circumstances, the diversification returns are similar. As to whether it is always better to rebalance a portfolio or not, the empirical evidence here is that it is not. Clearly points exist below, and above, the 45-degree equality line, implying that it is sometimes beneficial to rebalance, and sometimes it is not. We, therefore, conclude that equation (7) does not represent an upper bound for the diversification return of the non-rebalanced portfolios.

Table 2 summarizes the mean monthly geometric, strategic and diversification returns, in basis points, averaged over the 20-year period for the rebalanced and non-rebalanced portfolios.

Table 2: Mean monthly geometric, strategic and diversification returns: Rebalanced and Non-Rebalanced portfolios

	Market Cap		Equal Weight		Quality		Low Volatility		Value	
	Rebalance	Non-Rebalance	Rebalance	Non-Rebalance	Rebalance	Non-Rebalance	Rebalance	Non-Rebalance	Rebalance	Non-Rebalance
Geometric Return (bp)	53.68	53.33	75.86	71.78	65.13	64.69	61.19	59.29	61.76	59.14
Strategic Return (bp)	27.45	27.45	41.75	41.75	42.21	42.21	40.33	40.33	35.28	35.28
Diversification Return (bp)	26.24	25.88	34.11	30.03	22.92	22.48	20.85	18.96	26.48	23.85
Volatility (% per month)	4.96	4.80	5.43	5.25	4.72	4.61	4.54	4.42	5.04	4.87
2-Way Turnover (% pa)	77.44	9.40	82.13	28.88	88.68	29.75	72.75	13.75	84.63	31.16

Source: FTSE Russell. Data based on the FTSE USA Index between September 2000 and September 2020. Please see the end for important legal disclosures.

It appears that the differences in the averaged geometric return between rebalanced and non-rebalanced portfolios are small. Since the strategic returns are the same for rebalanced and non-rebalanced portfolios by definition, the small differences are due to variances in the diversification return. The small difference favors the diversification return of the rebalanced portfolios on average, but clearly from figure 1, we see this is not the case for every year over which the average has been performed. Indeed, it could be argued that once transaction costs are included into these results, the non-rebalance portfolios would have the advantage since their turnover figures are significantly lower.

For all portfolios, the larger contribution to the 20-year average geometric returns arises from the strategic return. Nevertheless, the diversification return contributes to a significant proportion of the geometric mean return. This is because the diversification component is always positive, whereas the strategic component may be positive or negative, and therefore has the possibility of “cancelling out” over the long term. We will examine this in more detail in the next section.

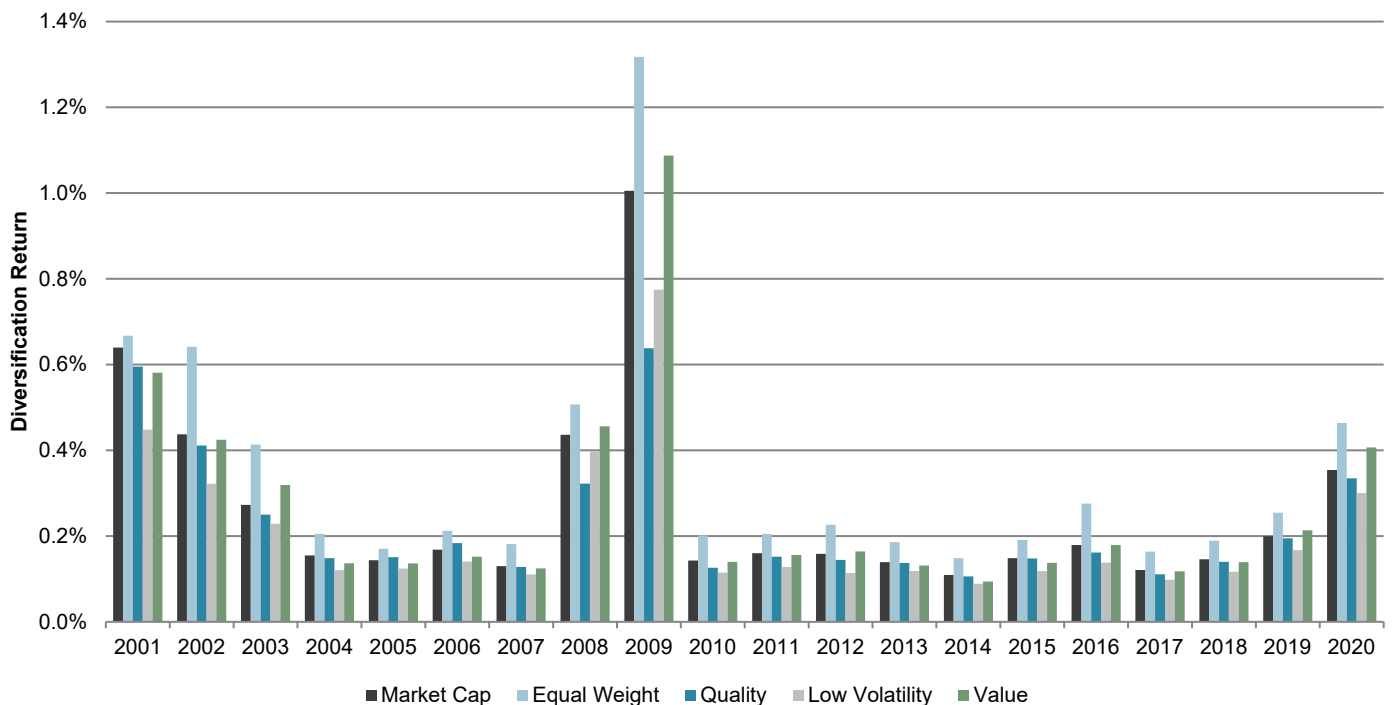
In summary, our analysis shows that there is little difference between the diversification return harvested by the rebalanced and non-rebalanced portfolios, except in certain exceptional years when a subset of stock returns, drive portfolio weights significantly apart.

4. What drives the difference in portfolio performance?

In this section, we decompose the rebalance portfolio’s returns into the strategic and diversification return components for each of our factor portfolios. Our aim is to determine their relative contribution to the monthly geometric mean return. We remark that we could have equally performed the same analysis on the non-rebalance portfolios and have drawn similar results and conclusions to those that we present here.

Figure 4 shows the diversification return for each of our rebalanced portfolios for each yearly period ending in September.

Figure 4: Diversification return through time: Rebalance portfolios

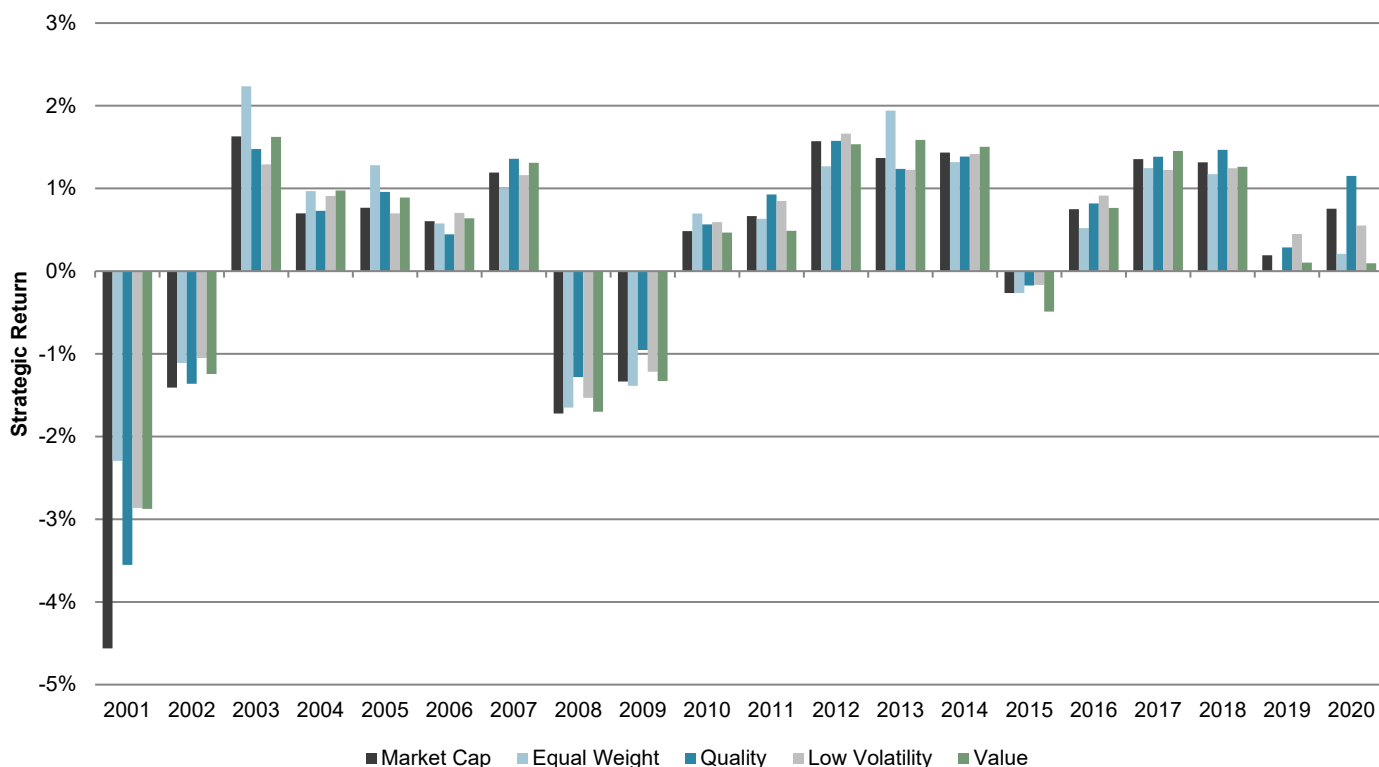


Source: FTSE Russell. Data based on the FTSE USA Index between September 2000 and September 2020. Please see the end for important legal disclosures.

Note that for most years, the diversification return is similar for each of the rebalance portfolios, and is positive, but small, being less than about 0.2%. However, during years of high volatility, 2001-2003, 2008-2009 and 2020, the diversification returns are not only bigger, but differ more significantly between portfolios. Equal weighting consistently has the highest diversification return, but it is noteworthy that market capitalization weighting, not normally associated with diversification, is not far behind.

In contrast figure 5 shows the strategic returns of each rebalance portfolio through time.

Figure 5: Strategic return through time: Rebalance portfolios

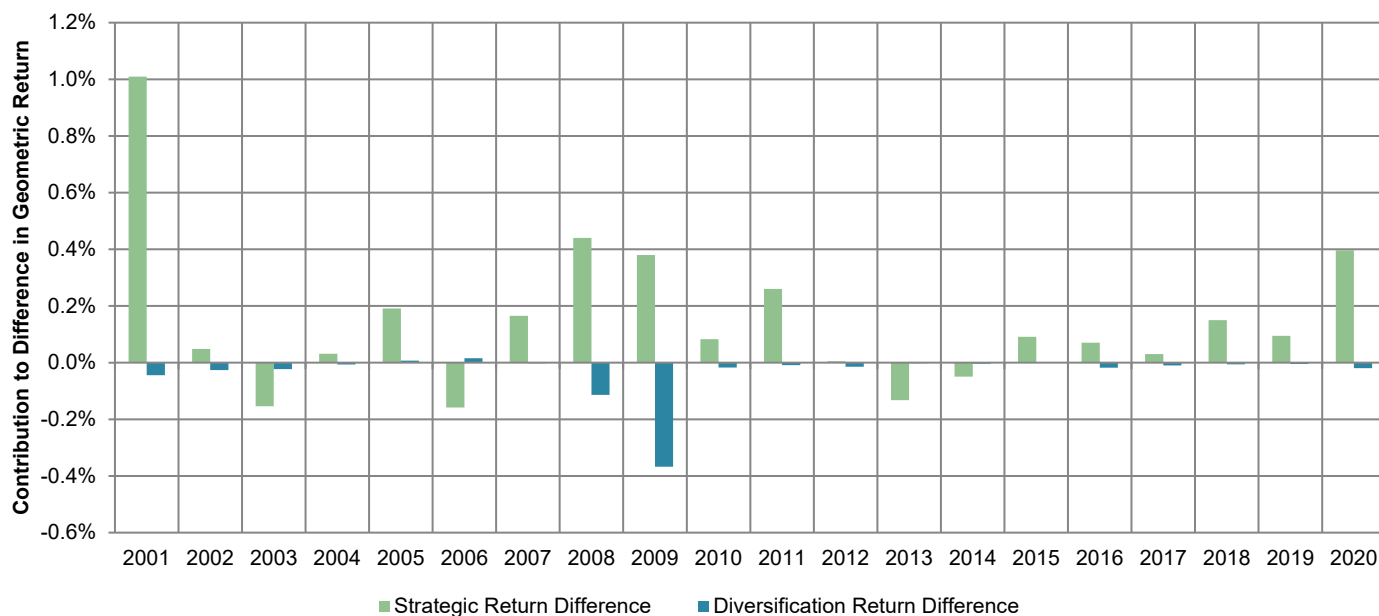


Source: FTSE Russell. Data based on the FTSE USA Index between September 2000 and September 2020. Please see the end for important legal disclosures.

Note that the strategic returns are significantly larger in magnitude than the corresponding diversification returns, being of the order of +/-1% in most years. Since the sum of the strategic return and the diversification return determines the geometric mean return, this graph implies that the strategic return is the dominant contributor to geometric return, apart from during the financial crisis of 2008-2009. However, as we have seen in the previous section, when averaged over the 20-year period, the contributions from the strategic and diversification returns, are closer in magnitude.

We can gain further insight by noting that performance is usually evaluated relative to the market return. As a proxy for this, figure 7 shows the differences between the strategic returns and diversification returns of the Quality rebalance portfolio and the market capitalization rebalance portfolio on a yearly basis.

Figure 6: Strategic and diversification return differences: Quality Rebalance portfolio vs Market Cap Rebalance portfolio



Source: FTSE Russell. Data based on the FTSE USA Index between September 2000 and September 2020. Please see the end for important legal disclosures.

It is clear that the difference between strategic returns dominates in most years. The contribution from the diversification returns approximately cancels out in every year, apart from 2008 and 2009. This can be explained by the fact that the diversification return is small, positive and available to both the Quality and the underlying market cap portfolios; therefore differences in diversification return are unlikely to be the main source of differences in the geometric returns of the two portfolios.

This result is emphasized in Table 3, which shows the time-averaged basis point difference in geometric mean, strategic and diversification returns between our rebalance factor and market capitalization portfolios.

Table 3: Average difference in geometric, strategic and diversification return of Factor and Market Cap Rebalance portfolios (bps)

Difference in bps	Equal Weight	Quality	Low Volatility	Value
Geometric Return	22.18	11.45	7.51	8.08
Strategic Return	14.30	14.76	12.89	7.84
Diversification Return	7.88	-3.32	-5.38	0.25

Source: FTSE Russell. Data based on the FTSE USA Index between September 2000 and September 2020. Please see the end for important legal disclosures.

Note that the results of Table 3 are easily derived from those of Table 2, by subtracting the results for a rebalanced market capitalization portfolio from each of the other rebalanced portfolios. It may be argued that, since the non-rebalanced market capitalization portfolio is naturally closer in weight to the usual FTSE USA Index benchmark, it is a more appropriate comparator. However, it is clear that subtracting the non-rebalanced market capitalization portfolio would make very little difference to the results in Table 3.

Only in the case of the equal weight portfolio does a sizable proportion of the positive difference in geometric mean return arise from the diversification return. For the Value portfolio, the positive contribution is very small. In the case of Quality and Low Volatility, the diversification return actually contributes negatively to the difference. In other words, on average, the diversification return for the rebalanced market capitalization weighted portfolio is greater than that of the Quality and Low Volatility rebalanced portfolios.

On the other hand, it is clear that the average differential strategic return contributions are positive, dominant and consistent with the expected factor premia payoffs. This is unsurprising since the strategic differences are directly driven by the active weights, which in turn have been designed to capture factor exposure.

Note that the significant difference between equal and market capitalization portfolio strategic returns found in this analysis contrasts results reported by Banner et al. [4]. They state that the strategic return is approximately the same for several “naïve” weighting schemes, so that differential performance is attributed solely to the diversification return. These naïve weighting schemes include capitalization weighting, equal weighting, random weighted, inverse random weighted and a “large overweight” scheme that weights in proportion to the square of market capitalization. This discrepancy may result from the fact that our analysis spans the period of 2000-2020, whereas their study takes place over the longer, but less recent period of 1964-2012. We assume that what characterizes their portfolios strategies as “naïve” is that no deliberate stock picking or exposure properties are sought after or assumed. This certainly contrasts our factor tilt portfolios that are explicitly designed for factor exposure.

Our results also seem to be at odds with recent research by Lin and Sanger [3]. They conclude that the bulk of smart beta performance arises through harvesting of diversification return through portfolio rebalancing. Again, this may be partly explained by their analysis being over the period 1972-2013, instead of 2000-2020. However, although they do perform a Fama-French regression analysis on portfolio returns to identify alpha associated with the diversification return, they also report factor betas that are positive and significant. Therefore, they actually demonstrate that factor exposures do influence the returns of these portfolios.

5. Conclusions

There has been considerable debate about the nature of a novel source of portfolio return, known as the “excess growth rate” since it was introduced by Fernholz and Shay in 1982 [1]. It has widely been described as a “rebalance premium” derived as a result of regular portfolio rebalance, or as a “diversification return” arising from volatility reduction due to portfolio diversification.

We believe these interpretations arise from the elegant formula derived for it by Fernholz and Shay [1], and later by Booth and Fama [2]. The formula expresses the diversification return as half the difference between the weighted average stock volatility of the constituents of a portfolio and portfolio volatility. Therefore, it contributes positively to the total geometric return of a portfolio, and can be directly understood as arising from the diversification benefits of holding a portfolio, rather than individual stocks. It would also appear that rebalancing is essential to harvest this return, since the expression is derived under the assumption that a portfolio is rebalanced back to a constant set of portfolio weights.

In this note, we have shown that a more general notion of diversification return—defined as the difference between the geometric return of a portfolio and a weighted sum of stock level geometric returns (or “strategic return”)—may be applied to both rebalanced and non-rebalanced portfolios. We have gone on to argue that, under circumstances where a non-rebalanced portfolio’s weights do not drift too far from their starting points, Fernholz and Shay’s formula valid for portfolios that are regularly rebalanced to constant weights, also yields a reasonable estimate for the non-rebalanced diversification return. We have then verified this empirically for various rebalanced and non-rebalanced factor portfolios.

The upshot of this analysis is the rather unsurprising result that, in general, the relative performance of a set of rebalanced portfolios compared to an equivalent set of non-rebalanced portfolios depends on market conditions. The strategic return component is common to both sets of portfolios, so any performance difference arises when Fernholz and Shay’s formula fails for the diversification return of the non-rebalanced portfolio. When this happens, either portfolio could outperform the other. Despite this, we find some evidence that longer term averages favor the rebalanced portfolio, but that such an advantage is small, and would probably be insufficient to cover the higher level of transaction costs incurred.

Nevertheless, we conclude that, when considering absolute geometric return of both rebalanced and non-rebalanced portfolios, the diversification return contributes a significant component.

However to assess the performance of factor portfolios, it is usual to calculate their performance *relative* to a chosen benchmark. On taking our market capitalization weighted portfolio as a proxy for this benchmark, we have seen that the diversification return contributions tend to “cancel out” since they are positive, small and similar in both the factor and benchmark portfolios. This leaves the dominant contribution to geometric return arising from differences in strategic returns. Note that the strategic return is directly related to the active weight of each of our factor portfolios, which have been specifically designed to capture positive active factor exposure. Therefore, the relative performance of our factor portfolios is driven by their active factor exposures, rather than the differential harvesting of diversification return. Similar results were found for factor portfolios in a study by Greene and Rakowski [10].

We hope that this analysis clarifies the subtle, and much misunderstood, concept of the diversification return and its contribution to portfolio performance.

6. Appendix

Note the familiar expression relating the portfolio return $R_{P,t+1}$ at time $t + 1$ to the individual stock weights at time t and stock return $r_{i,t+1}$ between time t and $t + 1$ is:

$$R_{P,t+1} = \sum_{i=1}^N W_{i,t} * r_{i,t+1} \quad (9)$$

Taking time averages of both sides of (2) gives:

$$\bar{R}_P = \frac{1}{T} \sum_{t=0}^{T-1} \sum_{i=1}^N W_{i,t} * r_{i,t+1} \quad (10)$$

Crucially when $W_{i,t} = W_{i,0}$ for all t , that is when the weights are rebalanced back to a constant set of starting weights, this equation simplifies to:

$$\bar{R}_P = \sum_{i=1}^N W_{i,0} * \bar{r}_i \quad (11)$$

In other words, the average portfolio return is equal to the weighted sum of the average stock returns when rebalanced weights are held at fixed values.

7. References

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